

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
International GCSE**

Centre Number

Candidate Number

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Monday 17 June 2019

Afternoon (Time: 2 hours)

Paper Reference **4PM1/01R**

**Further Pure Mathematics
Paper 1R**



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 - *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain **NO** credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
 - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times \text{slant height}$

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \frac{a}{1 - r}$ $|r| < 1$

Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{2!}x^2 + \dots + \frac{n(n - 1)\dots(n - r + 1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

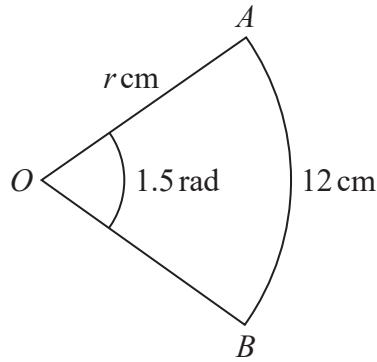


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows sector AOB of a circle with centre O and radius r cm. The angle AOB is 1.5 radians and the length of arc AB is 12 cm.

Calculate

(a) the value of r ,

(1)

(b) the area of the sector AOB .

(2)

(Total for Question 1 is 3 marks)



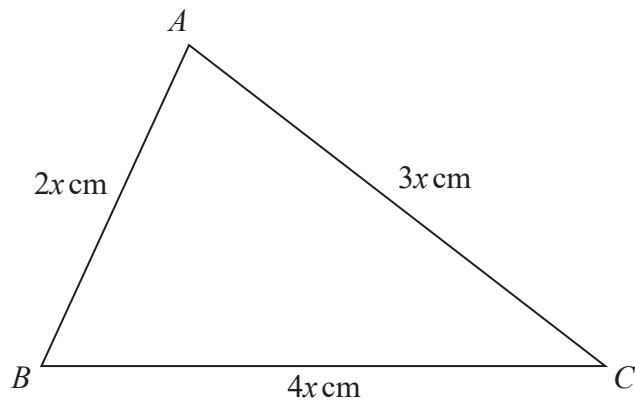


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Figure 2

Figure 2 shows triangle ABC in which

$$AB = 2x \text{ cm} \quad AC = 3x \text{ cm} \quad BC = 4x \text{ cm}$$

- (a) Show that $\sin ABC = \frac{3\sqrt{15}}{16}$ (4)

Given that the area of triangle ABC is $\frac{75\sqrt{15}}{64} \text{ cm}^2$

- (b) find the value of x . (2)



Question 2 continued

(Total for Question 2 is 6 marks)



3 (a) Write down the value of $\log_3 9$

(1)

(b) Solve the equation $\log_3 9t = \log_9 \left(\frac{12}{t} \right)^2 + 2$ where $t > 0$

Give your answer in the form $a\sqrt{b}$ where a and b are prime numbers.

(6)

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Question 3 continued

(Total for Question 3 is 7 marks)



4

$$f(x) = e^{3x} \sqrt{1+2x}$$

- (a) Show that

$$f'(x) = \frac{2e^{3x}(2+3x)}{\sqrt{1+2x}}$$

(4)

- (b) Find an equation of the normal to the curve with equation $y = f(x)$ at the point on the curve where $x = 0$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)



Question 4 continued

(Total for Question 4 is 10 marks)



- 5 A circle has radius $3r$ cm and area A cm 2

Given that the value of r increases by 0.05%

use calculus to find an estimate for the percentage increase in the value of A .

(5)

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Question 5 continued

(Total for Question 5 is 5 marks)



6 (a) Show that $\sum_{r=1}^n (4r - 3) = n(2n - 1)$ (3)

(b) Hence, or otherwise, find the least value of n such that $\sum_{r=1}^n (4r - 3) > 1000$ (3)

Given that $S_n = n(2n - 1)$, $t_n = (4n - 3)$ and that $18 + 3t_{n+7} = S_{n+4}$

(c) find the value of n . (4)





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Question 6 continued

(Total for Question 6 is 10 marks)



7 O, A, B and C are fixed points such that

$$\overrightarrow{OA} = 8\mathbf{i} - 6\mathbf{j} \quad \overrightarrow{OB} = 15\mathbf{i} - 6\mathbf{j} \quad \overrightarrow{OC} = 8\mathbf{i} + \mathbf{j}$$

- (a) Find \overrightarrow{BC} as a simplified expression in terms of \mathbf{i} and \mathbf{j}

(2)

- (b) Find a unit vector parallel to \overrightarrow{BC}

(2)

The point M is the midpoint of OA and the point N lies on OB such that $ON:NB = 1:2$

- (c) Show that the points M, N and C are collinear.

(4)



Question 7 continued



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Question 7 continued

(Total for Question 7 is 8 marks)



- 8 (a) Complete the table of values for $y = 2 + \ln(2x + 1)$ giving your answers to 2 decimal places.

(2)

x	0	0.25	0.5	1	1.5	2	3
y	2			3.10	3.39	3.61	

- (b) On the grid opposite, draw the graph of $y = 2 + \ln(2x + 1)$ for $0 \leq x \leq 3$

(2)

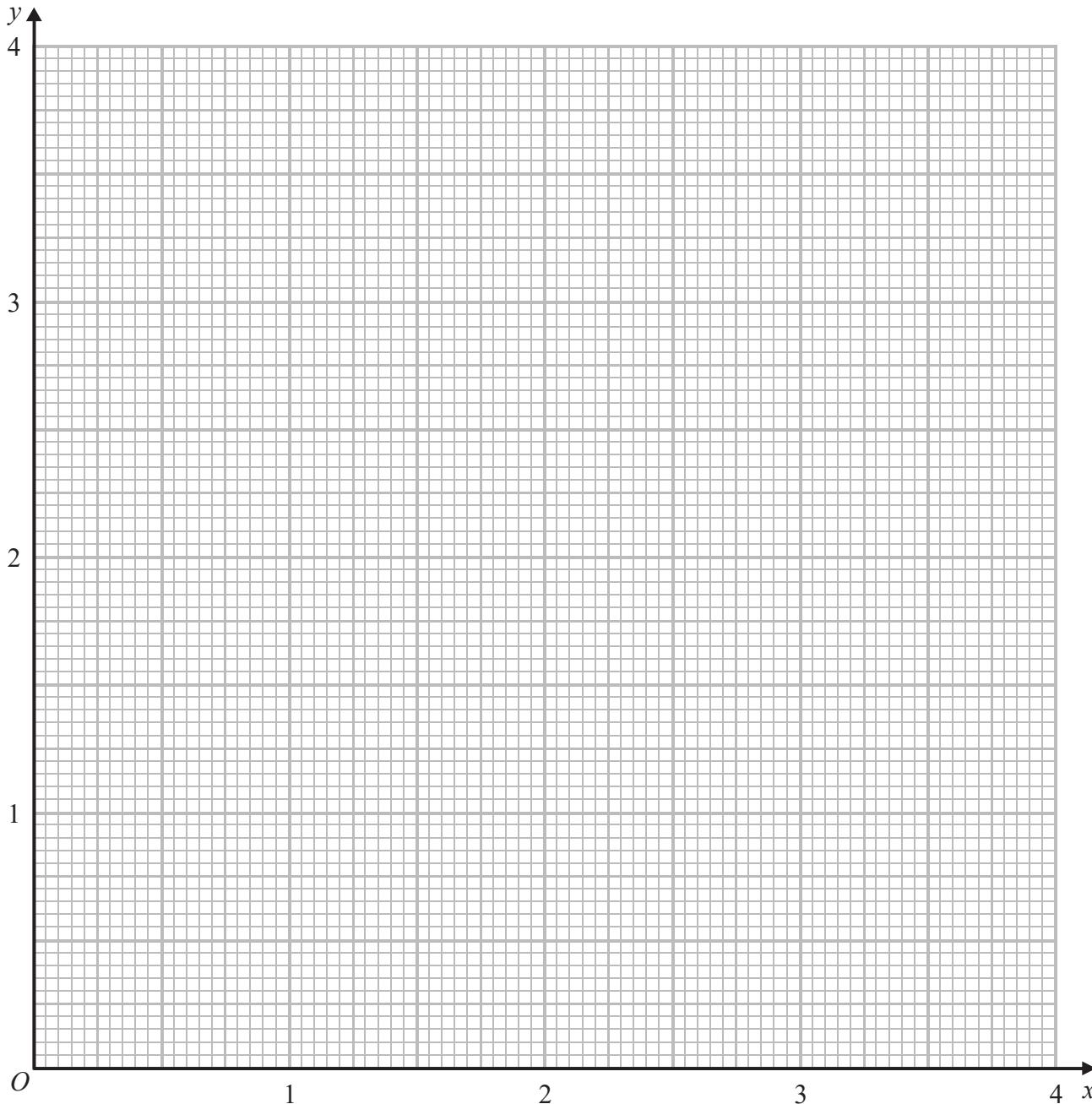
- (c) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $\ln(2x + 1) = 3x - 4$ in the interval $0 \leq x \leq 3$

(3)

- (d) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $e^{(6-x)} - (2x + 1)^2 = 0$ in the interval $0 \leq x \leq 3$

(4)



Question 8 continued

Turn over for a spare grid if you need to redraw your graph.



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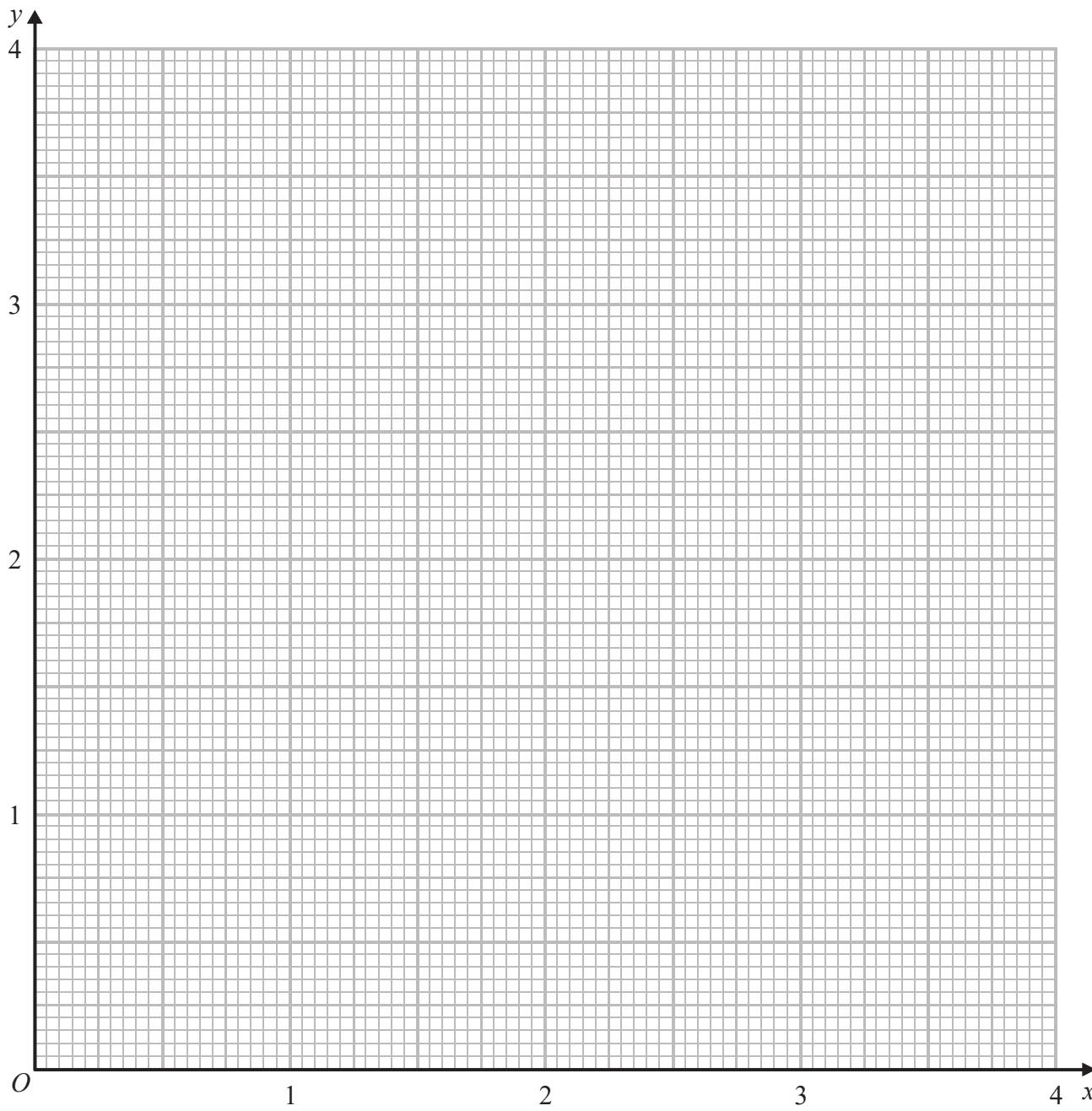
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Question 8 continued

Only use this grid if you need to redraw your graph.



(Total for Question 8 is 11 marks)



9

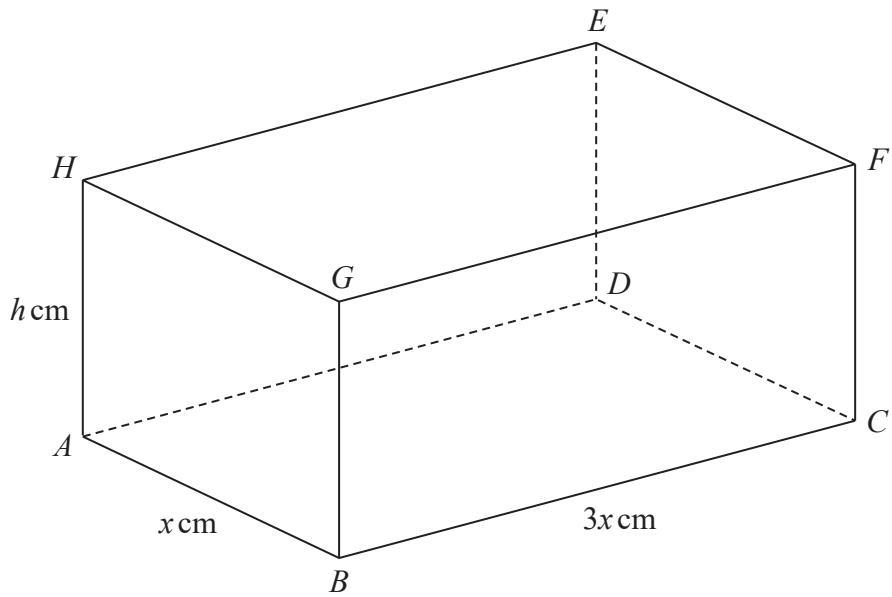


Figure 3

Figure 3 shows a solid cuboid $ABCDEFGH$

$$AB = x \text{ cm} \quad BC = 3x \text{ cm} \quad AH = h \text{ cm}$$

The volume of the cuboid is 540 cm^3

The total surface area of the cuboid is $S \text{ cm}^2$

(a) Show that $S = 6x^2 + \frac{1440}{x}$ (4)

Given that x can vary,

- (b) use calculus to find, to 3 significant figures, the value of x for which S is a minimum.
Justify that this value of x gives a minimum value of S . (5)

- (c) Find, to 3 significant figures, the minimum value of S . (1)



Question 9 continued

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Question 9 continued

(Total for Question 9 is 10 marks)



10

$$f(x) = 6x - x^2 \quad x \in \mathbb{R}$$

Given that $f(x)$ can be written in the form $D(x + E)^2 + F$ where D, E and F are integers,

- (a) find the value of D , the value of E and the value of F .

(3)

- (b) Find

- (i) the maximum value of $f(x)$,
(ii) the value of x for which the maximum occurs.

(2)

The curve C has equation $y = f(x)$

The curve S has equation $y = x^2 - 4x + 8$

The curve S intersects the curve C at two points.

- (c) Find the coordinates of each of these two points.

(4)

The finite region R is bounded by the curve C and the curve S .

- (d) Use algebraic integration to find the area of R .

(4)





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Question 10 continued

(Total for Question 10 is 13 marks)



11 The points A and B have coordinates $(-1, 3)$ and $(5, 6)$ respectively.

- (a) Find an equation for the line AB .

(2)

The point P divides AB in the ratio $2:1$

- (b) Show that the coordinates of P are $(3, 5)$

(2)

The point C with coordinates (m, n) , where $m > 0$, is such that CP is perpendicular to the line AB .

Given that the radius of the circle which passes through A , P and C is 5

- (c) find the value of m and the value of n .

(6)

The point D with coordinates (p, q) is such that the line AD is perpendicular to the line AB and the line DC is parallel to the line AB .

- (d) Find the value of p and the value of q .

(3)

- (e) Find the area of trapezium $ABCD$.

(4)



Question 11 continued



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Question 11 continued



Question 11 continued

(Total for Question 11 is 17 marks)

TOTAL FOR PAPER IS 100 MARKS

